The Anatomy of the Ten-Year Floating-Rate Retail JGB
―An Approach Based on the FRA Market Model―

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Abstract

The Ministry of the Treasury began to issue ten-year floating rate JGBs for retail purposes in March 2003, as part of the establishment of a funding diversification policy. Many security firms advertised these securities by means of recommendation letters that detailed the attractive characteristics of these products. However, to my knowledge, no research has yet evaluated the value of these products by means of an appropriate valuation model. The aim of this paper was to quantitatively clarify the attractive qualities of the ten-year floating-rate retail JGB, by means of a comparison with the 15-year floating-rate wholesale JGB, and with other, imaginary products. In the course of this study, I make use of the following powerful valuation tools: (1) a convexity correction through a numeraire change (Jamshidian, F.), (2) long-jump and very-long-jump approaches (Rebonato, R.), and (3) Kloeden and Platen's Runge-Kutta-like weak approximation. My major empirical findings are comprised of the following three results. First, the value of the ten-year floating-rate retail JGB is generally larger than that of the 15-year floating-rate wholesale JGB. Second, the price sensitivity of the former, with respect to the α BP that is subtracted from the base rate, is not monotonically decreasing. Third, the impetus of the second result is the trade-off, resulting from an increase in α BP, between the decrease in the projected future cash flow and the increase in the intrinsic value of the embedded American put option.

1. Introduction

The Japanese government has been issuing an enormous quantity of JGBs (Japanese government bonds) to finance its public investments; these investments have been set up with the aim of stimulating a Japanese economy that has been stagnant ever since the economic bubble burst (1986-1990). In order to facilitate the smooth and efficient functioning of the financing operation, the Ministry of the Treasury (MOT) has diversified the maturity and the product genre of its JGBs. In the course of implementing this diversification policy, the MOT began to issue 15-year floating-rate JGBs for wholesale purposes in 2000. In the recent low-interest-rate environment, this type of floating rate bond has come to be well accepted by many institutional investors, and the 15-year floating-rate JGB for wholesale purposes became one of the primary products in the JGB market. Observing that the 15-year floating-rate JGB for wholesale purposes was very popular among institutional investors, the MOT also began to issue ten-year floating-rate JGBs, for retail purposes, in March 2003.

Because the 15-year floating-rate wholesale JGB was traded by professional institutional investors in the primary and secondary markets, various papers suggesting evaluation methods for these products were published, primarily, by security firms. While existing papers on the 15-year floating-rate wholesale JGB are abundant, papers suggesting methods for the evaluation of the ten-year floating-rate retail JGB are scarce, despite the fact that numerous advertising pamphlets recommending these products have been distributed. One reason for this lack of research is that the evaluation of the ten-year floating-rate retail JGB is much more difficult than that of the 15-year floating-rate wholesale JGB, despite the fact that investors in the former tend to be individuals rather than professional institutional investors. Thus, the goal of this paper is to quantitatively clarify the nature of the ten-year floating-rate retail JGB.

To this end, I first review the standard industry convexity correction method used in the valuation of the 15-year floating-rate wholesale JGB from the standpoint of a numeraire change, based on the work of Jamshidian (1997). Second, the BGM model (Brace, Gatarek and Musiela (1997)), which is the market model used in the HJM (Heath, D.; Jarrow, R.; Morton, A.; (1992)) framework, and the long-jump and very-long-jump approaches (Rebonato R), as well as Kloeden and Platen's Runge-Kutta-like weak approximation, are briefly summarized. Third, the evaluation model for the ten-year floating-rate retail JGB, and the parameter estimation technique to be used in the model, are presented in detail. Fourth, several numerical examples that compare the valuation of the ten-year floating-rate retail JGB with those of the 15-year floating-rate wholesale JGB, and other, imaginary products, are used to clarify and illustrate the characteristics of the ten-year floating-rate retail JGB.

The organization of this paper is as follows. The second section presents the specifications of both the 15-year floating-rate wholesale JGB and the ten-year floating-rate retail JGB The third section provides the standard market model for the 15-year floating-rate wholesale JGB. The fourth section introduces the notations for, and the dynamics of, the forward rate and the swap rate. The fifth section reviews the framework of the FRA market model and risk-neutral drift approximations. The sixth section presents the evaluation model for the ten-year floating-rate retail JGB, as well as a detail of the model parameter estimation techniques. The seventh section provides several numerical examples, based on the proposed valuation model and its implications. The last section concludes the paper and suggests some directions for further research.

2． Specifications of the 15-year floating-rate wholesale JGB and the ten-year floating-rate retail JGB

In this section, the specifications of the 15-year floating-rate wholesale JGB and the ten-year floating-rate retail JGB are summarized, and the similarities and differences between the two are clarified. In the discussion that follows, I also note specific differences that interfere with the application of the convenient valuation formula for the wholesale JGB to the valuation of the retail JGB.

2.1 Specifications of the wholesale JGB

Maturity：15 years

Base Rate：(For the first coupon) the ten-year JGB yield in the ten-year JGB auction, just prior to the wholesale JGB auction.

(For other coupons) the ten-year JGB yield in the ten-year JGB auction, one month prior to the month to which the first date for the accrual calculation of the retail JGB belongs.

Coupon：Base Rate－αBP

Floor Level：0%

One-time-payment amount：Face value×Coupon÷100×0.5

Conditions for sale before maturity：Any time, at the market price.

Revenue from sale before maturity：the market price and the accrued interest.

2.2 Specifications of the retail JGB

Maturity：ten years

Base Rate：(For the first coupon) the ten-year JGB yield in the ten-year JGB auction, just prior to the wholesale JGB auction.

(For other coupons) the ten-year JGB yield in the ten-year JGB auction at one month prior to the month to which the first date for the accrual calculation of the retail JGB belongs.

Floor Level：0.05%

One-time-payment amount：Face value×Coupon÷100×0.5

Conditions for sale before maturity：Allowable after the second coupon payment date

Revenue from sale before maturity：face value + accrued interest－the two times the coupon amount just prior to the sale

Timing of payment receipt：within four business days of the date of sale

2.3 Similarities and Differences

2.3.1 Similarities

The two main similarities between the wholesale JGB and the retail JGB include:

(1) The coupon is dependent on the ten-year auction yield.

(2) Due to the floor on the coupon, an option floor is implicitly embedded.

2.3.2 Differences

The four primary differences between the wholesale JGB and the retail JGB include:

(1) The maturity of the wholesale JGB is 15 years, while that of the retail JGB is ten years.

(2) The α BP subtracted from the base rate in the derivation of the coupon of the wholesale JGB is decided at the wholesale JGB auction and depends on market conditions, while that of the retail JGB is currently fixed at 80BP.

(3) The floor of the wholesale JGB is 0%, while that of the retail JGB is 0.05%.

(4) The revenue from the sale of the wholesale JGB is the market price plus the accrued interest, while that of the retail JGB is its face value plus the accrued interest, less two times the coupon amount just prior to the sale. As explained in the next section, the American put option on the retail JGB is itself implicitly embedded in the retail JGB.

2.4 Why is the convenient valuation formula for the wholesale JGB not applicable to the valuation of the retail JGB?

In the wholesale JGB, there is no implicitly embedded American put option on the wholesale JGB itself, and thus the convenient sum formula of the floorlet type that will be introduced in Section 3 may be used in its valuation. However, as I mentioned in Section 2.2, the amount that is received from the sale of the retail JGB is not the market price plus the accrued interest but, rather, its face value plus the accrued interest, minus twice the coupon amount just prior to the sale. Thus, as pointed out in Section 2.3.2 (4), the American put option on the retail JGB is itself implicitly embedded in the retail JGB, and the valuation method resorted to in practice is some sort of simulation. Of course, one may evaluate the wholesale JGB based on the same simulation as that used for the evaluation of the retail JGB. In fact, in Section 7, I compare the value of the wholesale JGB, as derived by the convenient formula, with the same value derived using the simulation, to calculate the simulation error in various numerical examples.

3. The market model of the 15-year floating-rate wholesale JGB

The structure of the wholesale JGB is basically the same as that of the ten-year constant maturity swap with a floor. Thus, the valuation formula of the latter is applicable to the valuation of the former and simply involves the replacement of the ten-year swap yield with the ten-year JGB yield. In this section, I explain (1) the difference between the constant maturity swap rate and the usual swap rate, (2) the means by which the swap rate may be converted into a constant swap rate (The financial sector refers to this as a "convexity correction"), and (3) that the valuation formula of the wholesale JGB may be viewed as a modified version of the valuation formula of the ten-year constant maturity swap with a floor. The standard industry convexity correction method has been frequently described in the literature; examples include Yamashita (2001), Hull (2000), and Brotherton-Ratcliffe and Iben (1993), amongst others. However, this formula cannot be used directly in the valuation of the retail JGB because its valuation is based not on the formula but on a simulation. I briefly review the standard industry convexity correction method from the viewpoint of a numeraire change, as in Jamshidian (1997), to clarify the relation between the valuation of the wholesale JGB and that of the retail JGB.

3.1 The difference between the constant maturity swap rate and the usual swap rate

The constant maturity swap rate differs from the swap rate, because the constant maturity swap pays the amount stipulated by the rate in a different manner to that assumed by the rate. Adopting the base rate described in Section 2.2 as an example, I will explain this in detail. For the usual ten-year JGB, half of the yield is scheduled to be paid semiannually during the period between the time when the yield is decided (reset) and its maturity; for the wholesale JGB and the retail JGB, however, the payment of the reset yield occurs only a half-year later than the yield reset. In short, the natural numeraire for the usual ten-year JGB is the half-year-forward-start ten-year-period annuity numeraire, and the natural numeraire for the wholesale JGB and the retail JGB is the numeraire for the discount JGB, whose maturity is a half year. Thus, from the viewpoint of martingale theory, the difference between the constant-maturity JGB yield and the usual JGB yield comes from the difference in the numeraire used in the calculation of the expectation. In the conversion from the usual yield to the constant maturity yield, the financial industry usually uses the following convexity correction method, instead of the above-mentioned numeraire change.

3.2 Convexity Correction

3.2.1 Convexity Correction (1)

The first step of the standard industry convexity correction method is as follows. As an example, the i-year-forward ten-year-maturity JGB yield is examined. The price of the i-year-forward ten-year maturity JGB, as a function of the i-year-forward ten-year maturity JGB yield that is decided using the i-year-forward ten-year-period annuity numeraire, is reevaluated based on the numeraire of the i-year-maturity discount JGB, as in (2). Preceding the reevaluation, the price function of the i-year-forward ten-year-maturity JGB is approximated by the second-order polynomial of the i-year-forward ten-year-maturity JGB yield, as in (1). Strictly speaking, the numeraire change from the i-year-forward ten-year-period annuity numeraire to the numeraire of the i-year-maturity discount JGB should be applied, without approximation, to each cash flow payment made semiannually in the period between the time the yield is decided (reset) and its maturity. For convenience, however, the industry uses the second-order polynomial approximation. The yield and the price of the i-year-forward ten-year-maturity JGB, derived from the current JGB yield term structure, are $\bar{S}_i^{10}$ and $P\left(\bar{S}_i^{10}\right)$, respectively. When the yield is S_i^{10}, the price is expressed by $P\left(S_i^{10}\right)$.

The second-order polynomial approximation of $P\left(S_i^{10}\right)$ is

$$P\left(S_i^{10}\right) = P\left(\bar{S}_i^{10}\right) + P'\left(\bar{S}_i^{10}\right)\left(S_i^{10} - \bar{S}_i^{10}\right) + \frac{1}{2}P''\left(\bar{S}_i^{10}\right)\left(S_i^{10} - \bar{S}_i^{10}\right)^2. \quad (1)$$

Taking the expectation under the numeraire of the i-year-maturity discount JGB, one may derive equation (2):

$$\begin{aligned} E^i\left(P\left(S_i^{10}\right)\right) &= E^i\left(P\left(\bar{S}_i^{10}\right) + P'\left(\bar{S}_i^{10}\right)\left(S_i^{10} - \bar{S}_i^{10}\right) + \frac{1}{2}P''\left(\bar{S}_i^{10}\right)\left(S_i^{10} - \bar{S}_i^{10}\right)^2 \right) \\ &= P\left(\bar{S}_i^{10}\right) + P'\left(\bar{S}_i^{10}\right)\left(E^i\left(S_i^{10}\right) - \bar{S}_i^{10}\right) + \frac{1}{2}P''\left(\bar{S}_i^{10}\right)\left(E^i\left(\left(S_i^{10}\right)^2\right) - 2\bar{S}_i^{10}E^i\left(S_i^{10}\right) + \left(\bar{S}_i^{10}\right)^2\right) \end{aligned} \quad (2)$$

The left-hand-side of equation (2), $E^i\left(P\left(S_i^{10}\right)\right)$, turns out to be $P\left(\bar{S}_i^{10}\right)$, because it is the expected price of the i-year-forward ten-year-maturity JGB under the numeraire of the i-year-maturity discount JGB. (The payment of the i-year-forward ten-year-maturity JGB occurs at time i, and time i is also the maturity of the i-year-maturity discount JGB.) Consistent with the argument above, the next result comes from the usual transformation of equation (2), as in the aforementioned references.

$$E^i\left(S_i^{10}\right) = \bar{S}_i^{10} - \frac{1}{2}\frac{P''\left(\bar{S}_i^{10}\right)}{P'\left(\bar{S}_i^{10}\right)}\left(\bar{S}_i^{10}\right)^2\left(\exp\left(\sigma^2(T-t)\right) - 1\right) \quad (3)$$

3.2.2 Convexity Correction (2)

In the valuation of the wholesale and retail JGBs, an i-year-forward ten-year JGB yield under the numeraire of the $(i+1)$-year-maturity discount JGB is required. In the first step of the convexity correction, the i-year-forward ten-year JGB yield under the numeraire of the i-year-maturity discount JGB, $E^i\left(S_i^{10}\right)$, was derived, and this accomplished half of the desired goal. However, $E^i\left(S_i^{10}\right)$ must still be transformed into $E^{i+1}\left(S_i^{10}\right)$, and this transformation is the second step of the convexity correction. This transformation is carried out based on the i-year-maturity discount JGB $B_i(0)$, the $(i+1)$-year-maturity discount JGB $B_{i+1}(0)$, and the i-year-forward six-month LIBOR $\bar{S}_i^{0.5}$, as follows:

$$
\begin{aligned}
E^{i+1}\left[S_i^{10}\right] &= \frac{B_i(0)}{B_{i+1}(0)} E^i\left[S_i^{10}(t)\frac{B_{i+1}(t)}{B_i(t)}\right] \\
&= \left(1+\bar{S}_i^{0.5}\right)\left[E^i\left(S_i^{10}(t)\right)E^i\left(\frac{1}{1+S_i^{0.5}}\right) + Cov^i\left(S_i^{10}(t), \frac{1}{1+S_i^{0.5}}\right)\right] \\
&= \left(1+\bar{S}_i^{0.5}\right)\left[E^i\left(S_i^{10}(t)\right)\frac{1}{\left(1+\bar{S}_i^{0.5}\right)} + Cov^i\left(S_i^{10}(t), \frac{1}{1+S_i^{0.5}}\right)\right] \\
&= E^i\left(S_i^{10}(t)\right) + \left(1+\bar{S}_i^{0.5}\right) Cov^i\left(S_i^{10}(t), \frac{1}{1+S_i^{0.5}}\right)
\end{aligned}
\qquad (4)
$$

Thus, the second step of the convexity correction yields $\left(1+\bar{S}_i^{0.5}\right)Cov^i\left(S_i^{10}(t), \frac{1}{1+S_i^{0.5}}\right)$.

3.3 The valuation formula for the wholesale JGB

I first present the valuation formula for the floor option embedded in the wholesale JGB. The base rate used in the first coupon rate will have already been decided on the issuance date of the wholesale JGB and, once the αBP in the coupon calculation is known in the auction, the floor option comprises 29 floorlets, one for each coupon from the second to the thirtieth. (Of course, before the auction is finished, the floorlet for the first coupon also exists, due to the unknown αBP.) In the case of the wholesale JGB, when the coupon rate is less than 0% -- in other words, when the base rate is less than αBP, -- the intrinsic value of the floorlet is revealed. This means that the strike rate of each floorlet is αBP, when the underlying asset is the base rate. Thus, the floor option value FV, for a face value of one, is given by the next formula.

$$
\begin{aligned}
FV &= \frac{1}{2} DF_1 Max\left(\alpha/100 - S_0^{0.5}, 0\right) \\
&+ \frac{1}{2}\sum_{i=1}^{29} DF_{i+1}\left(-CMS_i^{10}\cdot N\left(-d_i - \sigma_i\sqrt{T_i}\right) + \alpha/100\, N\left(-d_i\right)\right)
\end{aligned}
\qquad (5)
$$

where,

DF_{i+1} : ($i+1$)-year discount factor

CMS_i^{10} : convexity corrected i-year-forward ten-year JGB yield

T_i : time to the reset date of the underlying base rate applied to the i th floorlet (year)

$\alpha/100$: Floor rate (%),

d_i : $\dfrac{\ln\left(100CMS_i^{10}/\alpha\right) - \frac{\sigma_i^2}{2}T_i}{\sigma_i\sqrt{T_i}}$, where σ_i is the implied volatility of the ten-year swaption that begins at the maturity of each floorlet.

$N(\cdot)$: cumulative probability density function of the standard normal distribution.

When the value of the wholesale JGB, aside from the floor option, is denoted by OV, then

$$OV = \frac{1}{2}\left(\sum_{i=1}^{29} DF_{i+1}\left(CMS_i^{10} - \alpha/100\right) + DF_{30}\right). \tag{6}$$

Thus, the valuation V for the wholesale JGB is given by following convenient formula:

$$\begin{aligned} V &= FV + OV \\ &= \frac{1}{2} DF_1 Max\left(\alpha/100 - S_0^{0.5}, 0\right) \\ &+ \frac{1}{2}\sum_{i=1}^{29} DF_{i+1}\left(-CMS_i^{10} \cdot N\left(-d_i - \sigma_i\sqrt{T_i}\right) + \alpha/100\, N\left(-d_i\right)\right) \\ &+ \frac{1}{2}\left(\sum_{i=1}^{29} DF_{i+1}\left(CMS_i^{10} - \alpha/100\right) + DF_{30}\right) \end{aligned} \tag{7}$$

In the wholesale JGB auction, the theoretically fair value of α BP is derived by fitting the above V to a face value of 100.

4. The dynamics and notations of the forward Rate and the swap Rate

Even though the forward JGB rate and the spot JGB yield are actually used in the valuation of the wholesale and retail JGBs, from this section to the sixth section, I will use, instead, the equivalent terminology for the swap market, because such dynamics and notations are much more popular in the swap market than in the JGB market.

4.1 Dynamics and notation of the Forward Rate

4.1.1 Notation of the forward rate

Let the initial time be time zero. The forward rates used in the valuation are six-month rates beginning six months forward, 12 months forward, 18 months forward, and so on. I introduce notation to express the dynamics of these forward rates. As a time scale, the sixth month becomes one unit of time. At time zero, I call the rate between one unit of time in the future and two units of time in the future the "first forward rate," and the rate between two units of time in the future and three units of time in the future the "second forward rate." I define forward rates from the first to the 49-th in the same manner and denote them by f_i, $(i = 1, \ldots, 49)$. In this notation, f_0 becomes the six-month spot rate. In this fashion, forward rates of every variety are defined according to i, and the values of these forward rates at time T_n are denoted by $f_i(T_n)$, $(i = 1, \ldots, 49, n = 0, \ldots, 49)$. T_0 indicates time zero, and T_{49} indicates time at 49 units of time in the future. Under this notation, as presented in Figure 1, at time zero there is one spot rate （ $f_0(T_0)$ ） and 49 forward rates （ $f_i(T_0),(i=1,\ldots,49)$ ） ; similarly, one unit of time in the future, there will be one spot rate （ $f_1(T_1)$ ） and 48 forward rates （ $f_i(T_1),(i=2,\ldots,49)$ ） . In the same manner, with the passage of one unit of time, the next forward rate becomes the spot rate, and the number of forward rates decreases by one. Thus, 48 units of time in the future, only one spot rate, $f_{48}(T_{48})$, and one forward rate, $f_{49}(T_{48})$, will exist.

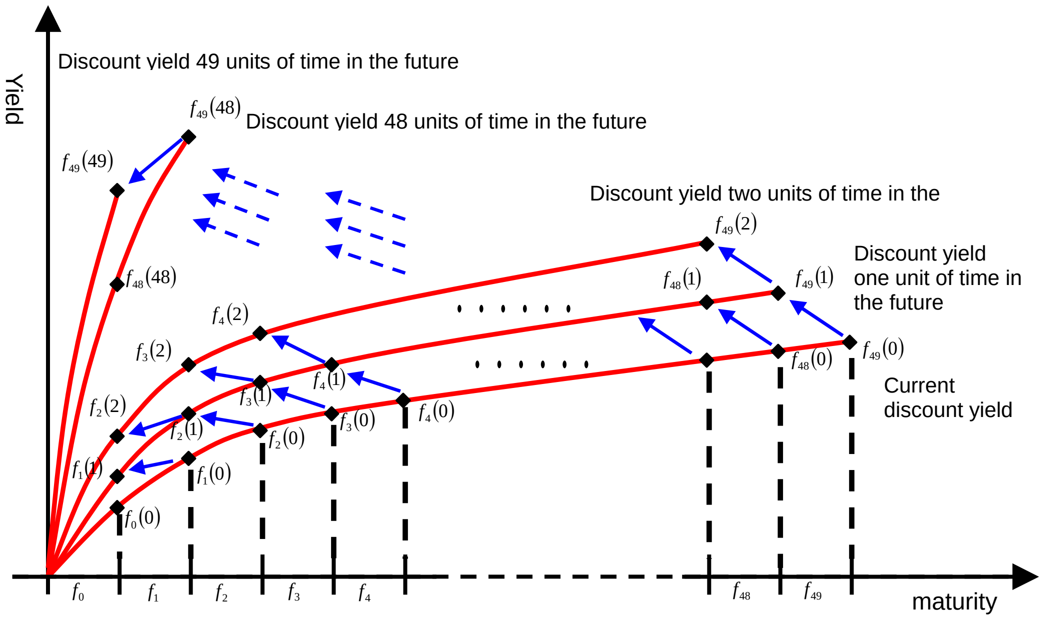

Figure1：Dynamics of the forward rates.

4.1.2 Dynamics of the forward rate

I assume the following dynamics of forward rate:

$$\frac{d\mathbf{f}(t)}{\mathbf{f}(t)} = \boldsymbol{\mu}(\mathbf{f},t)dt + \mathbf{S}(t)d\mathbf{w}_Q(t),$$

$$\boldsymbol{\rho}dt = d\mathbf{w}d\mathbf{w}^T \quad (dw_i dw_j = \rho_{ij}dt) \quad , \qquad (8)$$

where,

$\frac{d\mathbf{f}(t)}{\mathbf{f}(t)}$ ： time-dependent 49×1 column vector of percentage increments of the forward rates.

$\boldsymbol{\mu}(\mathbf{f},t)$ ： 49×1 column vector of drifts, which may depend both on time and on the forward rates themselves.

$d\mathbf{w}_Q(t)$ ： 49×1 column vector of correlated standard Brownian motions in measure Q, where Q

is the pricing measure implied by the chosen numeraire and $\mathbf{S}(t)$ is a real 49×49 diagonal matrix whose i^{th} element is equal to the instantaneous volatility σ_i of the i^{th} forward rate.

ρ ： 49×49 real symmetric instantaneous correlation matrix of the correlations among different forward rates

The stochastic differential equation (8) can be formally integrated to obtain the value of each forward rate $f_i(t)$ at time t:

$$f_i(t) = f_i(0)\exp\left[\int_0^t \mu_i(\{\mathbf{f}(u), u\}) - \frac{1}{2}\sigma_i^2(u)du + \int_0^t \sigma_i(u)dw_i(u)\right], \qquad (9)$$

where $f_i(t)$ and $f_i(0)$ indicate the values of the forward rates at time t and today (time zero), respectively.

4.1.3 Instantaneous volatility of the forward rate and Black volatility

The relationship between the time-dependent instantaneous volatility σ_i of the forward rate reset at time T_i and its Black (implied) volatility $\sigma_{Black}(T_i)$ is as given by the following equation:

$$\sigma_{Black}^2(T_i)T_i = \int_0^{T_i} \sigma_i^2(u)du \qquad (10)$$

4.2 Notation of the swap rate and its volatility based on the forward rate

4.2.1 Notation of the swap rate based on the forward rate

The swap rate is the fixed side rate exchangeable for the floating LIBOR in the swap agreement. Taking the i-year-forward ten-year-maturity swap rate S_i^{10} as an example, I now illustrate how the swap rate is expressed in terms of the forward rates introduced in the previous section. The value of the fixed side is $S_i^{10}/2\sum_{j=i}^{i+19} P(i, j+1) = S_i^{10}/A_i^{10}$ at *i* units of time in the future, where $P(i, j+1)$ is the price of the time ($j+1$)-maturity discount bond at i units of time in the future, and $A_i^{10} = 1/2\sum_{j=i}^{i+19} P(i, j+1)$. The value of the floating side is $1/2\sum_{j=i}^{i+19} f_j(T_i)P(i, j+1)$ at *i* units of time in the future. Because the fixed side value should be the same as the floating side value, the i-year-forward ten-year-maturity swap rate is expressed in terms of the forward rates as follows:

$$S_i^{10} = \sum_{j=i}^{i+19} w_j f_j(T_i), \qquad (11)$$

where $w_j = \dfrac{1/2\,P(i, j+1)}{1/2\sum_{j=i}^{i+19} P(i, j+1)} = \dfrac{1/2\,P(i, j+1)}{A_i^{10}}$.

4.2.2 The volatility of the swap rate in the forward-rate framework

I wish to point out that the volatility of the swap rate is not deterministic in the deterministic forward-rate volatility framework, and that it depends, instead, on the forward rate itself. As we saw above, the i-year-forward ten-year-maturity swap rate S_i^{10} at time t is expressed in terms of forward rates, with $T_i = t$ inserted into equation (11). The dynamics of the forward rate have already been given in Section 4.1.2, as represented by the stochastic differential equation. Thus, the dynamics of the i-year-forward ten-year-maturity swap rate S_i^{10} in the forward-rate framework are obtained through the use of the Ito formula. In particular, the instantaneous volatility $\sigma_i^{10}(t)$ of the swap rate is derived as follows:

$$\left(\sigma_i^{10}(t)\right)^2 = \frac{\sum_{j=i}^{i+19}\sum_{k=i}^{i+19}\left[\partial S_i^{10}/\partial f_j\right]\left[\partial S_i^{10}/\partial f_k\right] f_j(t) f_k(t) \rho_{jk}(t) \sigma_j(t) \sigma_k(t)}{\left[\sum_{j=i}^{i+19} w_j f_j(t)\right]^2} \tag{12}$$

As in the case of the forward rate considered in Section 4.1.3, the relation between the instantaneous swap volatility $\sigma_i^{10}(t)$ and its Black counterpart $\sigma_i^{10}(Black)$ is given by the next equation.

$$\left(\sigma_i^{10}(Black)\right)^2 T_i = \int_0^{T_i}\left(\sigma_i^{10}(u)\right)^2 du \tag{13}$$

5. Framework of the forward rate market model and risk-neutral drift approximations

5.1 Framework of the forward rate market model

With regard to the following implementation of the forward rate dynamics, which is based on the Monte Carlo simulation technique, one item is worthy of note. The objects of the simulation are forward rates for some number of units of time in the future, as illustrated in Figure 1. The abstract figure for the Monte Carlo simulation (a series of 49 random numbers generated from 49 independent standard normal distributions) is presented in Figure 2. In the actual valuation of the wholesale JGB, I only use the forward rates for up to 29 units of time in the future. The insight to be gained from Figure 2 is that the 1000 simulation values of the forward rate $f_2(2)$ are independent of the 1000 simulated values of forward rate $f_2(1)$. Moreover, any combination of the logarithms of these forward rates follows from the combined Gaussian distribution, though this is not illustrated in Figure 2. This attribute proves very useful in the valuation of derivatives, because the value of the forward rate at the time at which it is necessary to evaluate the derivative can be easily found by substituting the time into the bracketed expression in $f_i(\cdot)$. A sufficient condition for this characteristic is that both the drift $\mu_i(\{\mathbf{f}(u),u\})$ and the diffusion $\sigma_i(u)$ in equation (9) are deterministic. For further details about this requirement, the reader should refer to Nielsen (1999).

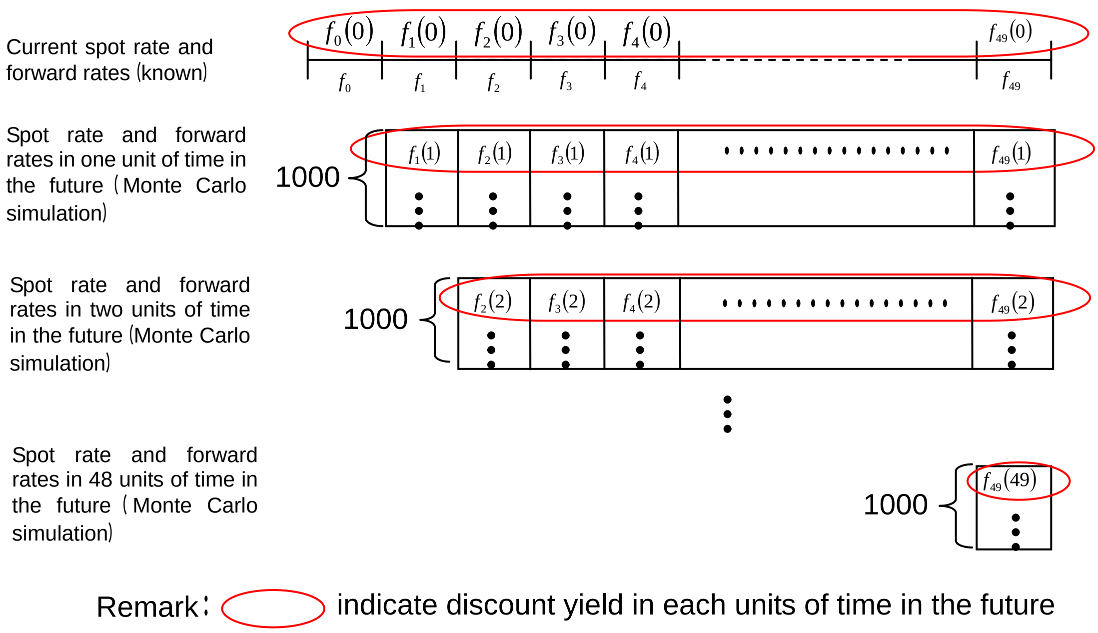

Figure 2 : Simulation of forward rates.

5.2 The risk-neutral forward rate and its applications

5.2.1 The risk-neutral forward rate

When the maturity T_{j+1} of the discount bond, which is the natural payoff of forward rate f_j, is chosen as a numeraire and is longer than the maturity of the i-th forward rate, the risk-neutral forward rate $f_i(t)$ is given by equation (14). For the derivation of equation (14), refer to Rebonato (2002).

$$\begin{aligned} f_i(t) &= f_i(0)\exp\int_0^t [\mu_i(\{\mathbf{f}\},u)]du + \sigma_i(u)dz_i(u) \\ &= f_i(0)\exp\int_0^t \left[-\sigma_i(u)\sum_{k=i+1}^{j}\frac{\sigma_k(t)\rho_{ik}(t)f_k(t)\tau_k}{1+f_k(t)\tau_k} - \frac{1}{2}\sigma_i^2(u)\right]du + \sigma_i(u)dz_i(u) \end{aligned} \tag{14}$$

Due to the presence of the unknown $f_k(t)$ in the integrand of equation (14), the risk-neutral forward rate $f_i(t)$ does not satisfy the condition given in Section 5.1. Thus, the logarithm of the risk-neutral forward rate no longer follows a Gaussian process, and the formerly convenient attribute of the simulation cannot be utilized. To overcome this obstacle, some sort of approximation must be applied to risk-neutral drift. The two main approximation techniques commonly used in this regard are as follows.

5.2.2 Approximation of risk-neutral drift (1)

One approximation technique consists simply of a replacement of the factor $f_k(t)\tau_k/[1+f_k(t)\tau_k]$, which is the stochastic variable, by the factor $f_k(0)\tau_k/[1+f_k(0)\tau_k]$, which is based on the initial value of forward rate $f_k(0)$. That is,

$$\begin{aligned}&\int_0^t \sum_{k=j+1}^{i} \frac{-\sigma_i(u)\sigma_k(u)\rho_{ik}(u)f_k(u)\tau_k}{1+f_k(u)\tau_k}du \\ &\cong \sum_{k=j+1}^{i} \frac{-f_k(0)\tau_k}{1+f_k(0)\tau_k}\int_0^t \sigma_i(u)\sigma_k(u)\rho_{ik}(u)du\end{aligned} \quad . \qquad (15)$$

Hull and White (2000) claimed this approximation to be 'innocuous,' but this can only be the case if the step size used in the Monte Carlo simulation is of the order of a forward rate tenor -- that is, three, six, or, sometimes, twelve months. However, in the pricing of the wholesale JGB, it is necessary to use a 14.5-year very-long-jump technique, and the crude approximation suggested above is by no means adequate. Actually, as the numerical examples in Section 7 indicate, the valuation of the wholesale JGB and the retail JGB based on this first approximation method does not meet the level of precision required by industry practitioners.

5.2.3 Approximation of risk neutral drift (2)

This second approximation method is the Runge-Kutta-like approximation introduced by Kloeden and Platen (1992); Hunter *et al.*(2001) applied it to the interest rate derivative arena. Kloeden and Platen showed that, given a stochastic process f with state-dependent drifts and volatilities $\mu(f,t)$ and $\sigma(f,t)$, respectively, and of known value $f(t)$ at time t, one can approximate its value at time $t+\Delta t$, that is, $f(t+\Delta t)$, as

$$\begin{aligned}f(t+\Delta t) \cong\ & f(t)+\frac{1}{2}\left[\mu(f,t)+\mu(\hat{f},t)\right]\Delta t \\ &+\frac{1}{4}\left[\sigma(\hat{f}^{+},t)+\sigma(\hat{f}^{-},t)+2\sigma(f,t)\right]\Delta z(t) \\ &+\frac{1}{4}\left[\sigma(\hat{f}^{+},t)-\sigma(\hat{f}^{-},t)\right]\left[\Delta z(t)-\Delta t\right]/\sqrt{\Delta t}\end{aligned} \qquad (16)$$

where, $\hat{f} = f(t)+\mu(f,t)\Delta t+\sigma(f,t)\Delta z(t)$,

$$\hat{f}^{+} = f(t)+\mu(f,t)\Delta t+\sigma(f,t)\sqrt{\Delta t}\,,$$

$$\hat{f}^{-} = f(t)+\mu(f,t)\Delta t-\sigma(f,t)\sqrt{\Delta t}\,.$$

Note that the term $\hat{f}$ is simply equal to the first-order Taylor approximation and could also be obtained using the first approximation method. Furthermore, the quantities $\sigma\left(\hat{f},t\right)$, $\sigma\left(\hat{f}^{+},t\right)$, and $\sigma\left(\hat{f}^{-},t\right)$ will all be equal to $\sigma(t)$, when the diffusive part is purely deterministic. All in all, the above approximation becomes

$$f(t+\Delta t) \cong f(t) + \frac{1}{2}\left[\mu(f,t) + \mu\left(\hat{f},t\right)\right]\Delta t + \sigma(f,t)\Delta z(t). \qquad (17)$$

In the pricing of the interest rate derivative whose maturity is T , equation (17) is actually utilized by assuming that $t=0$ and that $t+\Delta t = T$, because the initial time and the long-jump period are zero and T , respectively.

6. The valuation of the retail JGB and the technique used for parameter estimation

The technique for valuation of the retail JGB used in this section is based mainly on two approximations. One approximation was introduced in the derivation of the risk-neutral drift, as we saw previously. The other approximation is the evaluation of the Bermudan option, assuming that it is a good approximation of the American option embedded in the retail JGB. The time of exercise of the Bermudan option is restricted to a semiannual coupon payment date, while that of the American option may occur anytime until its maturity. Thus, the real value of the retail JGB is slightly higher than that indicated by a valuation technique employing a crude option approximation. If one increases the variety of the forward rate (a sampling frequency of not six months but, rather, of one month or one week) and also increases the number of exercise opportunities at the same time (from semi-annually to monthly or weekly), with some computational burden, then the value based on our technique does approach the true value. With regard to parameter estimation, a historical correlation matrix (which does not depend on time) is adopted as the matrix of correlations among the logarithms of the forward rates. The volatilities of the forward rates are estimated by ten-year swaption market prices, assuming that the volatilities of the forward rates are deterministic or, more precisely, constant in each period intervening between one swaption maturity and the next swaption maturity.

6.1 Technique for valuation of the retail JGB

The valuation of the retail JGB is based on the four steps described below.

Step 1 Simulation of the forward rates（refer to Figure 2, for up to 29 units of time in the future）

(1) Forty-nine independent series of 1000 random numbers are generated from the standard normal distribution.

(2) By way of a Cholesky decomposition of the historical correlation matrix of the forward rates (regarding the Cholesky decomposition, refer to Glasserman (2004), for example), the 49

series of independent random numbers are converted into 49 series of random numbers following a 49-variable correlated normal distribution.

(3) Taking the forward rate for one unit of time in the future as an example, the simulation technique can be explained as follows. The forward rate $f_i(1)$ is simulated based on equation (9), using the known value $f_i(0)$, and assuming that the volatility $\sigma_i(0 \to 1)$ of the forward rate f_i in the period is constant.（This estimation technique is illustrated in Section 6.2）The volatility of $f_i(1)$ is simulated simply by multiplying the i th series of random numbers in (2) by $\sigma_i(0 \to 1)\sqrt{0.5}$ （one unit of time is 0.5 years). Adding the volatility of $f_i(1)$ to the drift of $f_i(1)$, which is easily derived by means of the second approximation technique, as in Section 5.2.2 and Section 5.2.3, the forward rate for one unit of time in the future $f_i(1)$ may be simulated.

Step 2　Simulating the constant maturity swap rate

In the course of the valuation of the wholesale JGB and the retail JGB, the forward ten-year constant maturity JGB yield (in swap market wording, the constant maturity ten-year swap rate) needs to be simulated as the base rate. In the simulation, my choice of the numeraire is worthy of note. As mentioned in Section 5.2.1, the 25-year discount bond is adopted in this paper as a numeraire in the derivation of the risk-neutral drift of all the forward rates, and, of course, the swap rate in equation (11), which is based on the forward rates, is also derived using the same numeraire. The ten-year swap rate S_1^{10} for one unit of time in the future is derived based on equation (11) using $f_1(1),\cdots, f_1(20)$, and the ten-year forward rate S_2^{10} for two units of time in the future is also derived based on equation (11), but using $f_2(2),\cdots, f_2(21)$. In the same manner, the ten-year swap rate S_i^{10} for the i^{th} unit of time in the future is derived. The maturities of the wholesale JGB and the retail JGB are 15 years and ten years, respectively. Thus, the swap rates from S_0^{10} to S_{29}^{10} should be simulated for the evaluation of the wholesale JGB while, for the valuation of the retail JGB, simulation of only the swap rates from S_0^{10} to S_{19}^{10} is required. As I pointed out before, the ten-year swap rate S_i^{10} for the i^{th} unit of time in the future is here simulated using the 25-year discount bond （$B_n(t)$） as numeraire. However, the actual timing of payment S_i^{10} is $i+1$ units of time in the future. Thus, the natural numeraire for this swap rate is the maturity bond for the $(i+1)^{\text{th}}$ unit of time （$B_{i+1}(t)$）, and the numeraire of S_i^{10} must be changed from the 25-year discount bond （$B_n(t)$） to the maturity discount bond for the $(i+1)^{\text{th}}$ unit of time （$B_{i+1}(t)$） , as follows:

$$
\begin{aligned}
E^{i+1}\left(S_i^{10}(t)\right) &= \frac{B_n(0)}{B_{i+1}(0)} E^n\left(S_i^{10}(t)\frac{B_{i+1}(t)}{B_n(t)}\right) \\
&= \frac{B_n(0)}{B_{i+1}(0)}\left(E^n\left(S_i^{10}(t)\right)E^n\left(\frac{B_{i+1}(t)}{B_n(t)}\right) + Cov^n\left(S_i^{10}(t), \frac{B_{i+1}(t)}{B_n(t)}\right)\right), \quad (18) \\
&= E^n\left(S_i^{10}(t)\right) + \frac{B_n(0)}{B_{i+1}(0)} Cov^n\left(S_i^{10}(t), \frac{B_{i+1}(t)}{B_n(t)}\right)
\end{aligned}
$$

where n signifies the 25-year discount bond numeraire.

Therefore, in contrast to the convexity correction presented in Section 3.2, $\frac{B_n(0)}{B_{i+1}(0)} Cov^n\left(S_i^{10}(t), \frac{B_{i+1}(t)}{B_n(t)}\right)$ must be adjusted.

Step 3 Deciding the amount of the payment to be made at each payment date

The valuations of the wholesale JGB and the retail JGB are explained separately in this step, and in the next.

(Wholesale JGB)

If the ten-year JGB yield i units of time in the future is larger than α BP, then half of the difference is paid $i+1$ units of time in the future. Otherwise, nothing is paid at that time. Thus, the amount paid $i+1$ units of time in the future is $Max\left(1/2\left(S_i^{10} - 1/100\,\alpha\right), 0\right)$ %. Fifteen years in the future, at maturity, the notional 100 is also paid.

(Retail JGB)

As I mentioned in Section 6, in this valuation model, the retail JGB is assumed to be putable at each semi-annual coupon payment date, just like the Bermudan swaption.（As in the specification in Section 2.2, the time of exercise should be one year subsequent to its issuance.）The revenue received from the sale of the retail JGB before maturity is given by:

Face value＋accrued interest－two consecutive coupon payments made just prior to the sale

At the sale of the retail JGB i units of time in the future, the payment amount PP_i is given by

$$
\begin{aligned}
PP_i &= 100 + 1/2\,Max\left(S_{i-1}^{10} - 80/100, 0.05\right) \\
&\quad - 1/2\left(Max\left(S_{i-2}^{10} - 80/100, 0.05\right) + Max\left(S_{i-1}^{10} - 80/100, 0.05\right)\right).
\end{aligned}
\quad (19)
$$

Of course, the holder of the retail JGB receives all the coupon payments due prior to the sale.

Step 4 Evaluation of the wholesale JGB and the retail JGB based on the payment amount derived in Step 3

(Wholesale JGB)

As we saw in Step 3, the payoff $i+1$ units of time in the future is given by $Max\left(1/2\left(S_i^{10} - 1/100\,\alpha\right), 0\right)$ %, and I simulate 1000 paths for the payoff. I denote the accumulated reinvestment rate covering the period from $i+1$ units of time in the future to 25 years in the future by RI_i. RI_i is itself a stochastic variable, and 1000 paths of this are also simulated. Taking RI_1 and RI_2 as examples, their derivation is explained as follows. While S_1^{10} is composed of $f_1(1), \cdots, f_{20}(1)$, based on equation (11), as we saw in Step 2, RI_1 is composed of $f_2(1), \cdots, f_{49}(1)$, because it is the accumulated reinvestment rate covering the period from two units of time to 25 years in the future. RI_2 is composed of $f_3(2), \cdots, f_{49}(2)$, because it is the accumulated reinvestment rate covering the period from three units of time in the future to 25 years in the future. Each subsequent RI_i is derived in the same way. For each sample cash flow path, the total future value of the cash flow 25 years in the future, FVC^{25}, which is calculated using the reinvestment rate RI_i is given by the next equation.

$$FVC^{25} = \sum_{i=0}^{29} Max\left(1/2\left(S_i^{10} - \alpha/100\right), 0\right) RI_i + 100 \cdot RI_{29} \qquad (20)$$

Thus, the value of the wholesale JGB is derived by taking the expectation of equation (20) (taking the average of the 1000 simulated values of the FVC^{25}) and multiplying it by the price of the 25-year discount JGB, which is the numeraire in this evaluation.

(Retail JGB)

Because the framework of this research is based on the six-month forward rate, I approximate the value of the American option implicitly embedded in the retail JGB, which provides for exercise any time from one year after the issuance date to the maturity of the retail JGB, by the value of the corresponding Bermudan swaption, which provides for exercise at each six-month time interval from one year after the issuance date to the maturity of the retail JGB. Thus, the exact value of the retail JGB is thought to be slightly larger than the approximated value. In the case that one evaluates the American option using the simulation, regardless of the extent to which the granularity of the discretization is increased, the value of the American option is, in some sense, always approximated by some Bermudan type of swaption.（See Glasserman [2003], p.423）

In this framework, the American option can be exercised in each period of time from one year forward to the maturity of the retail JGB and, as this is the origin of the option, the payoff from its exercise is not the market value of the retail JGB, but the value given in Step 3. There is at least one optimal time at which to exercise the option, which may include the maturity of the retail JGB, and which maximizes the value of the retail JGB. There are five major methods for evaluating the American option. They are (1) parametric approximations, (2) random tree methods, (3) state-space partitioning, (4) stochastic mesh methods, and (5) regression-based methods. However, the evaluation methods used in the framework of Monte Carlo simulations are parametric approximations and regression-based methods. Two existing examples of work done on parametric approximations and regression-based methods are Andersen (2000), and Longstaff and Schwartz (2001), respectively. The latter study deals not only with interest rate derivatives but also with other varieties of financial derivatives. However, because our purpose is essentially the evaluation of the Bermudan type of swaption, based on the forward rate market model, I refer to the former study to evaluate the American option or, in other words, to construct the exercise boundary.

First, let $I(t)$ be an early exercise indicator function that equals one if early exercise is optimal at time t, and 0 otherwise. That is, with the notation introduced earlier, an early time of exercise τ^* is defined as follow:

$$\tau^* = \inf\{t \in \{T_2, T_3, \ldots, T_{19}\} : I(t) = 1\}.$$

The decision of whether or not to sell the retail JGB (to exercise the American option) at some time $T_i \in \{T_2, T_3, \ldots, T_{19}\}$ generally depends, in a complicated way, on the state of all forward rates $f_k(T_i)$, for $k = 2,3,\ldots,49$. As this "correct" exercise strategy depends on too many state variables to be feasible in a Monte Carlo setting, I first attempt to reduce the dimensionality of the exercise decision, following Andersen (2000), by postulating the following form of $I(t)$:

$$I(T_i) \approx f(TestVP_i(T_i), TestVP_{i+1}(T_i), \ldots, TestVP_{19}(T_i); p(T_i)), \tag{21}$$

where f is some specified Boolean function with a time-dependent vector of parameters $p(\bullet)$.
The $TestVP_i(T_i), TestVP_{i+1}(T_i), \cdots$, and $TestVP_{19}(T_i)$ in equation (21) are defined explicitly by the following formulas:

$$TestVP_i(T_i) = (PP_i(T_i) - HVP_i(T_i))RI_i^{50}(T_i) \cdot DF(T_{50}) / DF(T_i),$$

$$TestVP_{i+1}(T_i) = (PP_{i+1}(T_i) - HVP_{i+1}(T_i))RI_{i+1}^{50}(T_i) \cdot DF(T_{50}) / DF(T_i),$$

$$\cdots\cdots\cdots\cdots\cdots\cdots,$$

$$TestVP_{19}(T_i) = (PP_{19}(T_i) - HVP_{19}(T_i))RI_{19}^{50}(T_i) \cdot DF(T_{50}) / DF(T_i),$$

where $PP_j(T_i)$, for $j = i,\ldots,19$, is the value at time T_i of future cash flows that will be obtained when the retail JGB is sold at time j; $DF(T_{50})$ is the price at time zero of the 25-year discount JGB (the numeraire in this evaluation); and $DF(T_i)$ is the price at time zero of the time T_i maturity discount JGB. Furthermore, $RI_i^{50}(T_i)$ is the accumulated reinvestment rate, evaluated at time T_i, of the accumulated reinvestment rate covering the period from time T_i to maturity (50 units of time) of the numeraire discount JGB; $RI_{i+1}^{50}(T_i)$ is the rate, evaluated at time T_i, of the accumulated reinvestment rate covering the period from time T_{i+1} to maturity; and $RI_{19}^{50}(T_i)$ is the rate, evaluated at time period 19, of the accumulated reinvestment rate covering the period from the 19th unit of time to maturity. Finally, $HVP_i(T_i)$, $HVP_{i+1}(T_i)$, $\cdots$, and $HVP_{19}(T_i)$ are the values at time T_i of future cash flows that will be obtained when the retail JGB is held from T_i to maturity, from T_{i+1} to maturity,…, and from T_{19} to maturity, respectively. These may be evaluated as follows:

$$
\begin{aligned}
&HVP_i(T_i)\\
&= \sum_{j=i}^{20} Max\left(\frac{1}{2}\left(S_j^{10}(T_i)-\frac{\alpha}{100}\right),0.05\right)\exp\left(-0.5\sum_{k=i}^{j} f_k(T_i)\right)+100\exp\left(-0.5\sum_{k=i}^{20} f_k(T_i)\right),
\end{aligned}
$$

$$
\begin{aligned}
&HVP_{i+1}(T_i)\\
&= \sum_{j=i+1}^{20} Max\left(\frac{1}{2}\left(S_j^{10}(T_i)-\frac{\alpha}{100}\right),0.05\right)\exp\left(-0.5\sum_{k=i}^{j} f_k(T_i)\right)+100\exp\left(-0.5\sum_{k=i}^{20} f_k(T_i)\right),
\end{aligned}
$$

$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots,$

$$
\begin{aligned}
&HVP_{19}(T_i)\\
&= \sum_{j=19}^{20} Max\left(\frac{1}{2}\left(S_j^{10}(T_i)-\frac{\alpha}{100}\right),0.05\right)\exp\left(-0.5\sum_{k=i}^{j} f_k(T_i)\right)+100\exp\left(-0.5\sum_{k=i}^{20} f_k(T_i)\right).
\end{aligned}
$$

Common to all specifications of f is the boundary condition that is the exercise condition at the last time of exercise. More specifically,

$$
\begin{aligned}
I(T_{19}) &= \begin{cases} 1 & if \quad TestVP_{19}(T_{19}) > 0, \\ 0 & if \quad otherwise, \end{cases} \\
&= \begin{cases} 1 & if \quad PP_{19}(T_{19}) > HVP_{19}(T_{19}), \\ 0 & if \quad otherwise, \end{cases}
\end{aligned}
\tag{22}
$$

which simply states that the retail JGB is to be sold (the American option is to be exercised) at T_{19} (the last possible date), if and only if the present value of the cash flow resulting from the sale is larger than that resulting from retention.

In the specification of f , I adopt the first exercise approximation strategy given in Andersen (2000).

Exercise approximation strategy:

$$I(T_i)=\begin{cases}1 & if \quad H(T_i) > HVP_i(T_i), \\ 0 & if \quad otherwise.\end{cases} \tag{23}$$

In this specification, the retail JGB is sold when the present value of the cash flow obtained from retention until maturity, $HVP_i(T_i)$, falls below the time-dependent barrier $H(T_i)$, or the early exercise boundary.

Next, I derive the time-dependent early exercise boundary $H(T_i)$. $H(T_i)$ is characterized as the function that maximizes the value of the American option, given the exercise criterion in equation (21). Thus, $H(T_i)$ is derived by a backward-induction algorithm used to determine the maximum value of the retail JGB in each time period. Given equation (22), the value of $H(T_{19})$ is known, and the problem of determining the value of $H(T_{18})$ is reduced to a one-dimensional optimization procedure that seeks to maximize the expected cash flow from time T_{18} to maturity. $H(T_{18})$ is computed by means of the following numerical search. First, some initial value of $H(T_{18})$ is chosen. Second, when each $HVP_{18}(T_{18})$ simulated by the Monte Carlo method exceeds the initial value of $H(T_{18})$, $HVP_{18}(T_{18})$ is adopted; otherwise, it is replaced with $PP_{18}(T_{18})$ and multiplied by $RI_{18}^{50}(T_{18})\cdot DF(T_{50})/DF(T_{18})$. Taking the expectation of this gives the present value of the expected cash flow for the retail JGB after time T_{18}, when the exercise boundary at time T_{18} is the initial value of $H(T_{18})$. Thus, one has then only to search numerically for the value of $H(T_{18})$ that maximizes the present value of the expected cash flow of the retail JGB after time T_{18}. Once the early exercise boundary $H(T_{18})$ is obtained, in the same fashion, one may determine the early exercise boundary $H(T_{17})$, in order to solve a one-dimensional optimization procedure for the maximization of the expected cash flow from time T_{17} to maturity. Repeating this backward-induction algorithm up to $H(T_2)$, one may determine all of the early exercise boundaries $H(T_i)$, for $i = 2,\ldots,19$.

Once the early exercise boundaries $H(T_i)$, for $i = 2,\ldots,19$, have been determined, based on the above procedure, one may determine the early time of exercise τ^* corresponding to each cash flow path simulation. If the retail JGB is sold at optimal exercise time τ^*, the total value of future cash flow, FVP_i^{25}, that is generated by the retail JGB, and is reinvested from the time of the occurrence of each payment until the maturity of the numeraire 25-year discount JGB, is given by the following formula:

$$FVP_{\tau^*}^{25} = \sum_{j=0}^{\tau^*-1} Max\left(1/2\left(S_j^{10} - \alpha/100\right), 0.05\right) RI_j + PP_{\tau^*} RI_{\tau^*-1} \qquad (\tau^* = 2,\ldots,19) \tag{24}$$

$$FVP_{20}^{25} = \sum_{i=0}^{19} Max\left(1/2\left(S_i^{10} - \alpha/100\right), 0.05\right) RI_i + 100 \cdot RI_{19}$$

(the case of retention until maturity)

Once 1000 sample path values of the $FVP_{\tau^*}^{25}$ have been derived, based on equation (24), the valuation of the retail JGB is obtained by taking their expectations and by multiplying them by the price of the numeraire 25-year discount JGB.

6.2 Estimation of the model parameters

To evaluate the wholesale JGB and the retail JGB based on this model, the instantaneous volatility $\sigma_i(t)$ ($S(t)$ in Section 4.1.2) of each forward rate $f_i(t)$, and the matrix of correlations ρ_{ij} among the forward rates, must be estimated. I adopt a historical correlation matrix, which does not depend on time, as the matrix of correlations among the logarithms of the forward rates. The volatilities of the forward rates are estimated by fitting the ten-year swaption volatilities, based on the forward rate market model, to their corresponding ten-year implied swaption volatilities. In the course of parameter estimation and numerical evaluation, the deterministic instantaneous volatility $\sigma_i(t)$ of each forward rate $f_i(t)$ is assumed to be constant for each unit of time. Thus, in the estimation of $\sigma_i(t)$, one must estimate 29 constant instantaneous volatilities, which are represented by $\sigma_i(0 \to 1), \sigma_i(1 \to 2), \ldots, \sigma_i(28 \to 29)$. (The expressions in parentheses indicate the units of time to which the constant volatilities pertain.) The above estimation of the constant volatility at each unit of time involves fundamentally the same procedure as the estimation of the following constant volatilities: $\sigma_i(0 \to 1)$, $\sigma_i(0 \to 2)$, …, $\sigma_i(0 \to 29)$. Because the constant volatilities actually used in the simulation are $\sigma_i(0 \to 1), \sigma_i(0 \to 2), \ldots, \sigma_i(0 \to 29)$, the technique used to estimate these is given by the following two steps.

Step 1 Technique for estimating constant volatilities for the full period

In Step 1, I adopt the same approximation technique that was used in Section 5.2.2. In this step, the volatility of each forward rate f_i is assumed to be the same constant over the full period. That is, $\sigma_i(0 \to 1), \sigma_i(0 \to 2), \ldots, \sigma_i(0 \to 29)$ are all equivalent (and are denoted by σ_i), and for $i \geq 30, \sigma_i = \sigma_{29}$. Thus, in Step 1, 29 constant volatilities for the full period, such as from σ_1 to σ_{29}, are estimated, based on mathematical programming. The equation used in the estimation is equation (13). As the observation data for the left-hand-side of equation (13), $\sigma_i^{10}(Black)$, or the implied volatility of the i^{th} time unit maturity ten-year swaption, is adopted. (There are 29 varieties of swaption maturities, which range from one unit of time in the future to 29 units of time in the future.)

A problem arises, however, because the integrand of equation (13) is equation (12), and equation (12) is not directly numerically integrable, because equation (12) includes the state variable f_i and the partial derivative of the swap rate with respect to the state variable f_i. To tackle this problem, even though the approximation granularity is not particularly suited, I apply the same approximation as in technique (1), in Section 5.2.2, to the integrand; that is, I replace the time-dependent variables and their functions with their values at the initial time. I also capture $\partial S_i^{10} / \partial f_j$ and $\partial S_i^{10} / \partial f_k$ in equation (12) as the formal partial derivatives of equation (11), and write them as w_j and w_k, respectively. In summary, the right-hand-side of equation (12) is approximated by

$$\frac{\sum_{j=i}^{i+19}\sum_{k=i}^{i+19} w_j(0) w_k(0) f_j(0) f_k(0) \rho_{jk}\sigma_j\sigma_k}{\left[\sum_{j=i}^{i+19} w_j(0) f_j(0)\right]^2}.$$

Thus, the mathematical programming setup for the estimation is as follows.

Objective Function： $\underset{\sigma_1,\cdots,\sigma_{29}}{Min} \sum_{i=1}^{29} \varepsilon_i^2$

Constraints： $\sigma_i > 0$ （$i = 1,\cdots,29$）,

$$\text{where } \varepsilon_i = \left(\left(\sigma_i^{10}(Black)\right)^2 - \frac{\sum_{j=i}^{i+19}\sum_{k=i}^{i+19} w_j(0) w_k(0) f_j(0) f_k(0) \rho_{jk}\sigma_j\sigma_k}{\left[\sum_{j=i}^{i+19} w_j(0) f_j(0)\right]^2} \right).$$

Step2 Technique for the estimation of constant volatilities for each period

In this step, I introduce the factor $h(T_i)$, which is common to every particular volatility σ_i of forward rate f_i, and is dependent on each period, to estimate the constant volatility in each period, based on the constant volatility in the full period. Using this factor, one may derive the constant volatility of forward rate f_i in each period, $\sigma_i(0 \to 1)$, $\sigma_i(0 \to 2)$, … , $\sigma_i(0 \to 29)$, as $\sigma_i(0 \to 1) = h(T_1)\sigma_i$, $\sigma_i(0 \to 2) = h(T_2)\sigma_i$, …, $\sigma_i(0 \to 29) = h(T_{29})\sigma_i$, by multiplying the constant volatility for the full period σ_i by the period-dependent factor $h(T_1), h(T_2), \ldots, h(T_{29})$. The procedure for estimating the factor $h(T_i)$ resorts to a numerical search via simulation, based on the valuation model under discussion. As in Step 2 of Section 6.1, the ten-year swap rate *i* units of time in the future, S_i^{10}, is simulated by way of approximation technique (2), as given in Section 5.2.3. (This approximation technique is far more precise than technique (1).) The simulated value of $S_i^{10}(T_i)$ is dependent on the unknown factor $h(T_i)$, because the volatility of forward rate f_i includes the unknown factor $h(T_i)$, as $\sigma_i(0 \to T_i) = \sigma_i h(T_i)$. (Refer to Figure 3.) Thus, one has only to search for the factor $h(T_i)$ to make the left-hand side of equation (22) equal to the observed market data on its right-hand side.

$$Var\left(\ln\left(\frac{S_i^{10}(T_i)}{S_0^{10}}\right)\right) = \left(\sigma_i^{10}(Black)\right)^2 T_i \tag{22}$$

Repeating the search 29 times, and adopting the 29 implied volatilities of the ten-year swaptions whose maturities range from one unit of time to 29 units of time, one may determine the 29 values of the factor $h(T_i)$, for $i = 1,\ldots,29$, and complete the estimation of the constant volatility for each period.

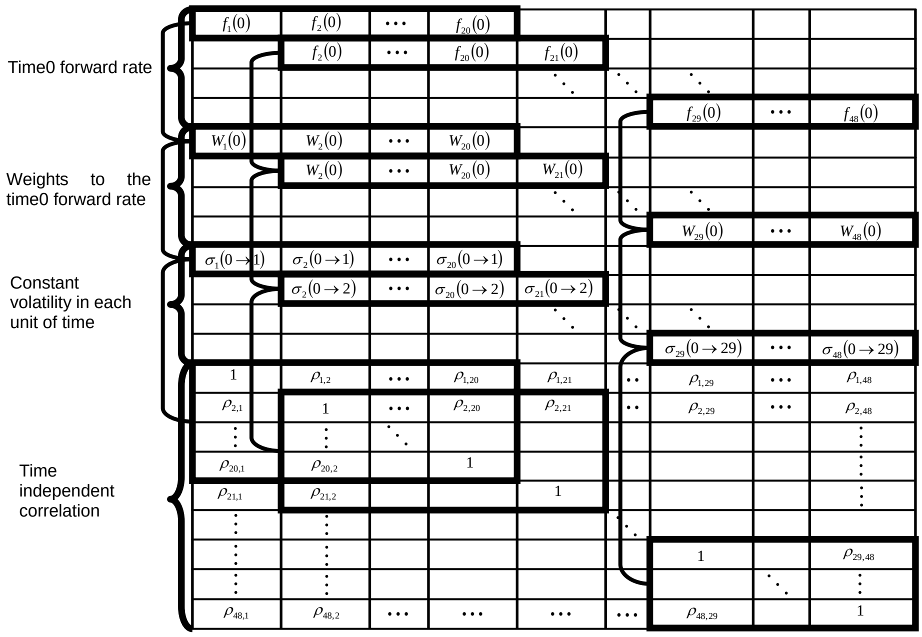

Figure３：Elements of volatility and correlation of swap rate.

7. Numerical Examples

The primary purpose of this section is to examine in a realistic setting the difference in approximation precision between approximation technique (1) and approximation technique (2). More precisely, I examine the extent to which the prices of at the money floorlets and caplets that have maturities ranging from one unit of time to 29 units of time differ from their BS prices. The second objective is to evaluate the value of the retail JGB and to examine in depth the origin of its value by comparing it with the wholesale JGB and various other similar imaginary products. Furthermore, the sensitivity of each product with respect to αBP, as in the case of the wholesale JGB, as well as the distribution of the optimally timed exercise of the American put option, are also analyzed.

7.1 Data

Spline-estimated discount JGB yields covering the period from 1/4/2000 to 12/29/2003 are used as the JGB yield data. In this numerical analysis, the values of the retail JGB from the first issuance to the third issuance are examined and, in Figure 4, three JGB yield curves corresponding to the three issuance dates 3/10/2003 (the first issuance), 4/10/2003 (the second issuance) and 7/10/2003 (the third issuance) are provided. In Figure 4, the low yield level, the middle yield level, and the high yield level are for 4/10/2003, 3/10/2003 and 7/10/2003, respectively. A close examination of Figure 4 reveals that the difference between the yield level on 4/10/2003 and the yield level on 3/10/2003 clearly appears to be more than 8 year sectors, and that the yield curve of the sectors on the latter date is a little steeper than that on the former date. Furthermore, the yield curve on 7/10/2003 is far steeper than those for the other two dates.

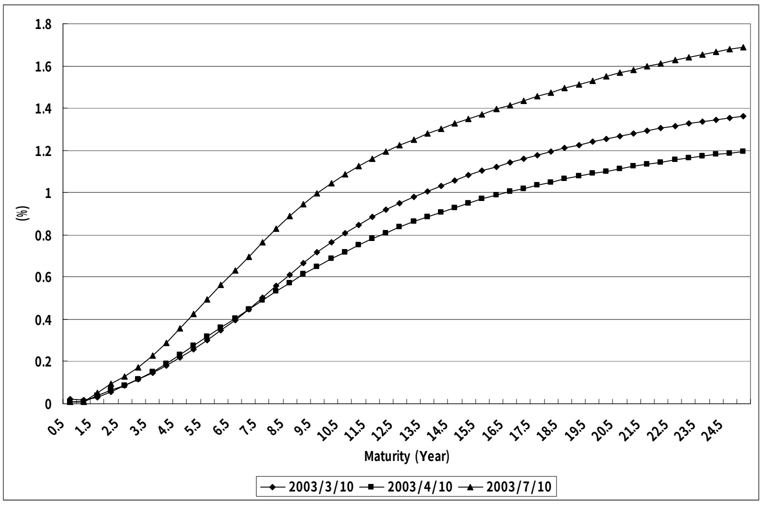

Figure 4：Three JGB yield curves corresponding to the three issuance dates 3/10/2003 (the first issuance), 4/10/2003 (the second issuance) and 7/10/2003 (the third issuance).

The volatilities of the ten-year swaption, whose maturities range from one unit of time to 29 units of time, are shown in Figure 5 for these three dates. The term 'historical', used in Figure 5, refers to the standard deviation of the i-year-forward ten-year JGB yield, derived from the discount JGB yield data. On 3/10/2003, for all maturities, the implied volatilities were approximately the same as their historical counterparts. On 4/10/2003, the implied volatilities were larger than the historical volatilities in the less-than-8-year maturities, and this difference was largest for the shorter maturities. Due to the huge sell-offs that occurred in June 2003, the implied volatility curve for 7/10/2003 takes on an extreme shape. The implied volatilities for the short maturity sectors were extremely large, whereas those in the longer maturity sectors were even smaller than the historical volatilities, and this makes the implied volatility curve extremely convex.

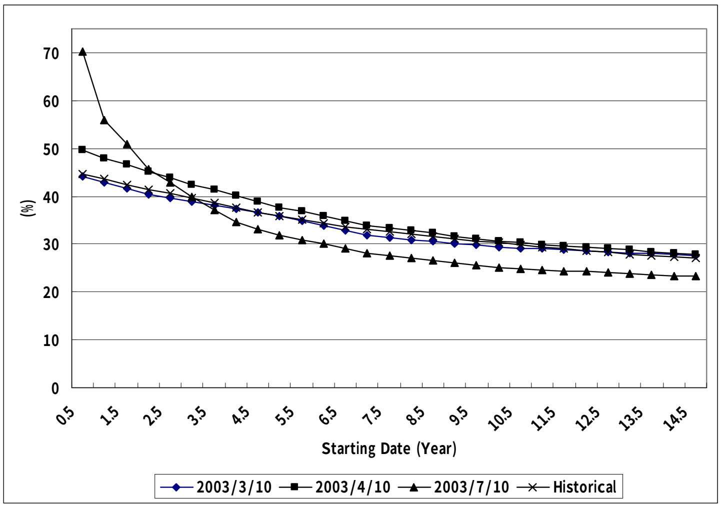

Figure5：The volatilities of the ten-year swaption, whose maturities range from one unit of time to 29 units of time corresponding to the three issuance dates 3/10/2003 (the first issuance), 4/10/2003 (the second issuance) and 7/10/2003 (the third issuance).

The historical matrix of correlations among forward rates is provided in Figure 6. The correlations among the forward rates, whose starting times were more than five years, were much higher than those among the forward rates, with starting times of less than five years, and the correlation among the forward rates with starting times greater than 18 years approached one.

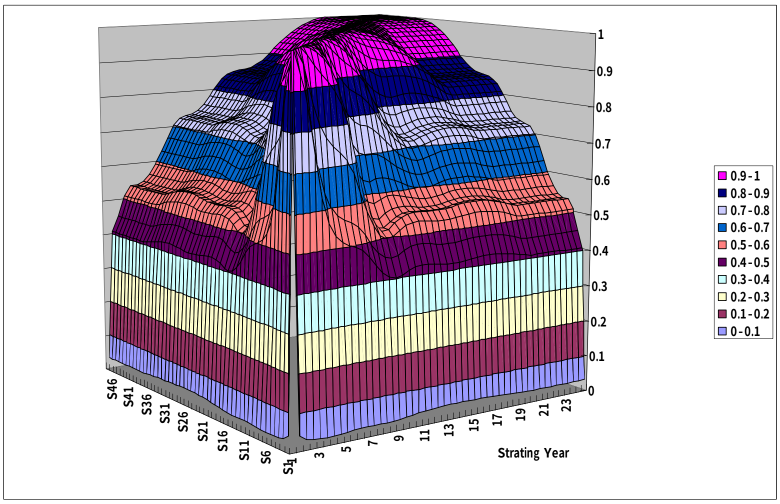

Figure6：The correlation of forward rates.

7.2 Estimation results for the constant volatility in each period and a comparison of approximation methods (1) and (2)

The estimated constant volatilities for each period of forward rates on the three issuing dates are shown in Figure 7. In accordance with the magnitude of the ten-year implied swaption volatilities, the estimated constant volatilities on 4/10/2003 were slightly larger than those on 3/10/2003, and the estimated constant volatilities for maturities of less than four years on 7/10/2003 were extremely large, whereas those with a more than ten-year maturity on 7/10/2003 were the smallest of those for the three issuing dates.

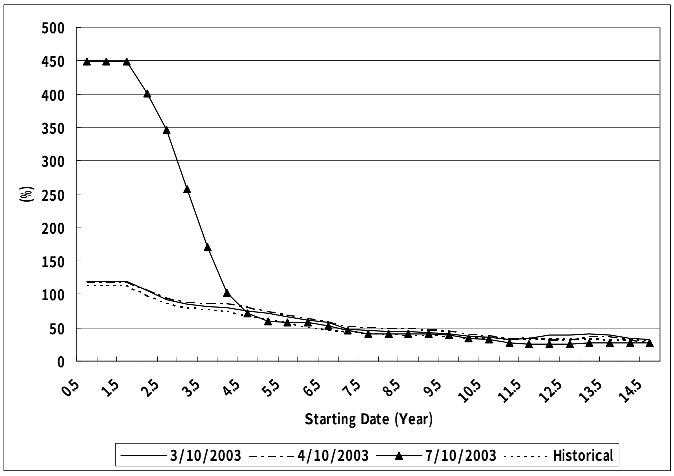

Figure 7 : The estimated constant volatilities for each period of forward rates on the three issuing dates.

The deviations of the simulation-based values of at the money floorlets and caplets, whose maturities ranged from a period of one unit to a period of 29 units, from those of the Black-Scholes formula on the three issuance dates, 3/10/2003, 4/10/2003 and 7/10/2003, are shown in Figures 8 through 10. In each figure, the label "BS" indicates the Black-Scholes prices of ATM floorlets and caplets, and these are provided as a reference from which to judge the relative magnitudes of the simulation-based pricing errors. "Caplet1" and "Floorlet1" mean, respectively, the caplet and floorlet price discrepancies between the simulation-based prices derived using the first risk-neutral drift approximation technique, and the Black-Scholes prices. "Caplet2" and "Floorlet2" indicate, respectively, the caplet and floorlet price discrepancies between the simulation-based prices derived using the second risk-neutral drift approximation technique, and the Black-Scholes prices.

Since the ATM forward option is being examined, the caplet price error and the floorlet price error should be almost the same at any maturity. However, at any issuance date, the simulation-based caplet prices derived via the first risk-neutral drift approximation are far larger than the BS prices, while the simulation-based floorlet prices derived using the first risk-neutral drift approximation are, in general, close to the BS prices. As the payoff function in Step 3 of Section 6.3 indicated, the simulation-based evaluation model for the retail JGB is on the side of caplet valuation. In other words, the values of the wholesale JGB and the retail JGB are basically the sums of the values of their respective caplets. Also, in the context of this valuation, the 25-year discount JGB is used as a numeraire and, in the evaluation, each cash flow increment is reinvested, up to the maturity of the numeraire discount JGB. The fact that the simulation-based caplet prices were far larger than the BS prices may indicate that the simulated reinvestment rates were higher than the exact reinvestment rates. When we see relatively large pricing errors, as for Caplet 1 in Figures 8 through 10, even in only one fraction of a cap, the wholesale JGB and the retail JGB cannot be evaluated with appropriate precision.

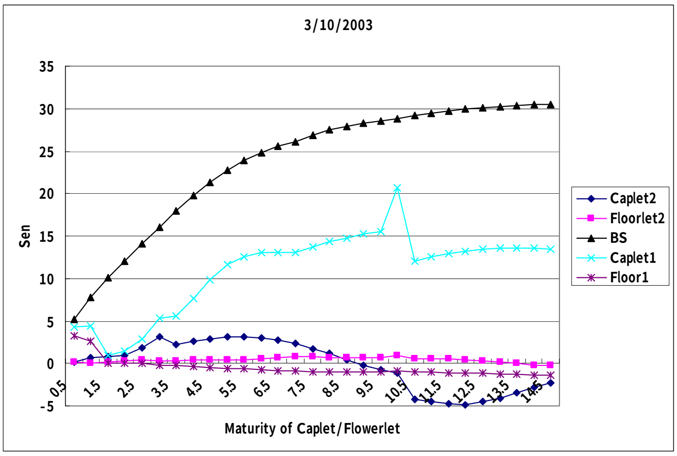

Figure8：The deviations of the simulation-based values of the money floorlets and caplets, whose maturities ranged from a period of one unit to a period of 29 units, from those of the Black-Scholes formula on the issuance dates 3/10/2003.

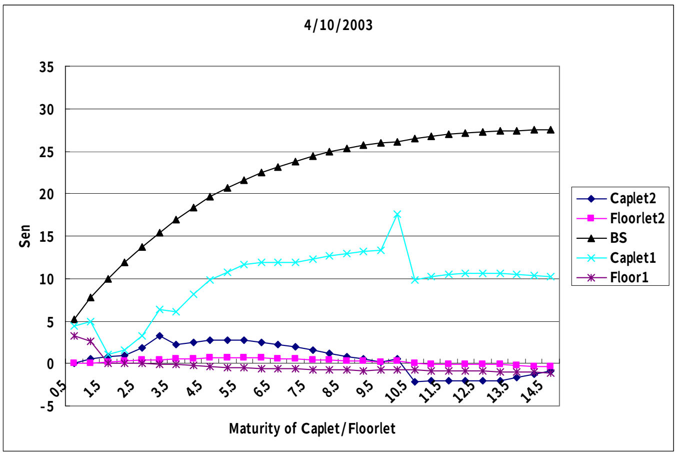

Figure9：The deviations of the simulation-based values of the money floorlets and caplets, whose maturities ranged from a period of one unit to a period of 29 units, from those of the Black-Scholes formula on the issuance dates 4/10/2003.

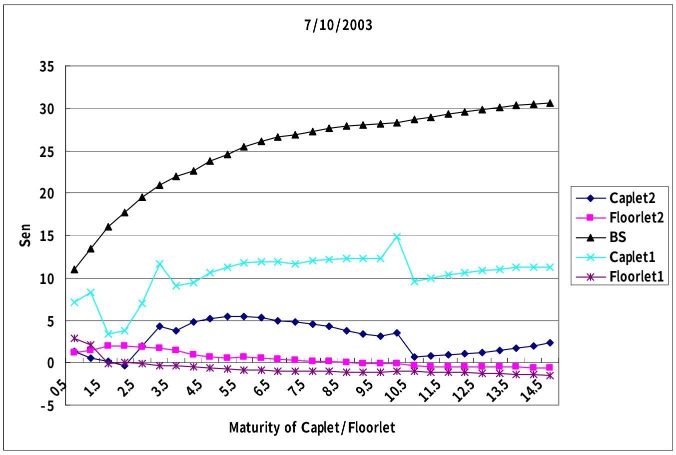

Figure10：The deviations of the simulation-based values of the money floorlets and caplets, whose maturities ranged from a period of one unit to a period of 29 units, from those of the Black-Scholes formula on the issuance dates 7/10/2003.

In contrast to the results derived using the first risk-neutral drift approximation, at any issuance date, the simulation-based caplet prices derived using the second risk-neutral drift approximation were quite close to the BS prices, and all of the differences in price were less than 5 Sen. Except for those for the issuance date 7/10/2003, the signs of the price deviations were not the same, but were dependent on the maturities of the caplet, and were, in fact, positive for maturities of less than ten years and negative for maturities greater than ten years. For the floorlets, all of the simulation-based prices, except those for maturities of less than three years at the issuance date 7/10/2003, were almost the same as the BS prices; furthermore, even for maturities of less than three years at the issuance date 7/10/2003, the price deviations were only around 2 Sen. Therefore, by adopting the second risk-neutral drift approximation, I was able to evaluate the wholesale JGB and the retail JGB with reasonable precision.

7.3 Anatomy of the ten-year floating rate retail JGB

To facilitate an examination of which factors influenced the value of the retail JGB, the valuations of the following six products, including the wholesale JGB and the retail JGB at the three issuance dates, 3/10/2003, 4/10/2003, and 7/10/2003, are compared in Table 1: (1) the valuation of the wholesale JGB, based on the formula given in Section 3; (2) the valuation of the wholesale JGB, based on the simulation-based model (using the same αBP as that used in (1)); (3) the valuation of the ten-year floating rate JGB, with a 0% floor and without the American option JGB, based on the simulation-based model (also using the same αBP as that used in (1)); (4) the valuation of the ten-year floating rate JGB, with a 0.05% floor and without the American option JGB, based on the simulation-based model (also using the same αBP as that used in (1)); (5) the valuation of the retail JGB, based on the simulation-based model (also using the same αBP as that used in (1));and (6) the valuation of the retail JGB, based on the simulation-based model（using 80BP as αBP）. Each value expressed in Table 1 corresponds to a face value of 100.

Table1：The valuations of the following six products, including the wholesale JGB and the retail JGB at the three issuance dates, 3/10/2003, 4/10/2003, and 7/10/2003, are compared.

	(1) FRN15Y Alpha	(1) FRN15Y BS	(2) FRN15Y Model	(3) FRN10Y Model	(4) FRN10Y+5BP Model	(5) Personal-Alpha Model	(6) Personal-80BP Model
3/10/2003	63	100.00	99.58	100.99	101.04	103.74	105.67
			-0.42	1.40	0.06	4.16	6.08
4/10/2003	54	99.99	99.96	101.05	101.12	103.54	109.06
			-0.03	1.09	0.07	3.58	9.10
7/10/2003	66	100	102.24	102.82	102.84	103.61	103.77
			2.24	0.57	0.03	1.37	1.53

Alpha in (1) is expressed in BP. There are two reasons for presenting item (2) as given above, both of which relate to the magnitude of the deviations of caplet prices derived from the simulation-based model in relation to those based on the Black-Scholes formula, as discussed in Section 7.2. First, it is important to know how much the simulation-based valuation of the wholesale JGB, which has its cash flow composed of the collection of many caplets, differs from the exact value of the same valuation derived by the BS formula in Section 3. Second, this item is useful as a basis for comparison. Because the valuation of the retail JGB is derived from the simulation-based model, in the comparison of this valuation to that of the wholesale JGB, since the same simulation-based model is not used for both bonds, it cannot be determined whether the discrepancy between the two values comes from the difference in the specification of the products or from the difference in the valuation models; therefore, the goal of identifying the factors influencing the value of the retail JGB cannot be attained. Item (3), above, is presented to allow examination of the relationship of the maturity factor to the valuation of the retail JGB. (Notice that the maturity of the wholesale JGB is 15 years, while that of the retail JGB is 10 years.) Similarly, item (4), above, is presented so that the impact of the floor level on the value of the retail JGB may be examined. (Notice that the floor level of the wholesale JGB is 0%, while that of the retail JGB is 0.05%.) Finally, item (5) is presented so that the extent to which the factor αBP subtracted from the base rate influences the value of the retail JGB may be analyzed. (Notice that the αBP of the wholesale JGB is decided at auction, while that of the retail JGB remains constant at 80BP.)

Examination of Table 1 reveals the extent to which the simulation-based price of the wholesale JGB deviated from the price derived by the BS formula. On 3/10/2003 and 4/10/2003, the model prices were lower than the BS prices by 42 Sen and 3 Sen, respectively, while on 7/10/2003, the model price was higher than the BS price by 2 Yen 24 Sen. In the current wholesale JGB market, an error of around 50 Sen is thought to be acceptable in practice, and so the valuations based on the simulation-based model were quite accurate for 3/10/2003 and 4/10/2003. As was seen in Figure 10 in Section 7.2, due to the fact that the model-based caplet prices for all maturities were higher than the BS formula prices, even if I had adopted the second risk-neutral-drift approximation approach, on 7/10/2003, the model price would have been far higher than the BS price. For a more precise evaluation for this date, a more elaborate approximation, such as the application of the second risk-neutral-drift approximation after a division of the period to maturity of the wholesale JGB, is necessary.

To examine the effect of the maturity factor, I compare the value of (2) with the value of (3). On three issuance dates, 3/10/2003, 4/10/2003 and 7/10/2003, the values of (3) were higher than the values of (2) by 1 Yen, 40 Sen; 1Yen, 9 Sen; and 57 Sen; respectively. An explanation of these deviations simply by means of the shape of the yield curve would not be precise, because these deviations include the deviations coming from the floor options. However, most of the deviations do seem to be explained by the shape of the yield curve. Due to the 15-year maturity of the wholesale JGB, the expected future cash flow depends, in the main, on the shape of the yield curve from the tenth year to the 25th year, and so the value of (2) depends on the curve shape; whereas, due to the ten-year maturity of the retail JGB, the expected future cash flow depends, in the main, on the shape of the yield curve from the tenth year to the 20th year, and so the value of (3) likewise depends on the curve shape. Thus, I divide the yield curve beyond the tenth year into two parts, one ranging from year ten to year 20, and one ranging from year 20 to year 25, and I attempt to determine which part is steeper by means of a comparison of the yield curves on 3/10/2003 and 7/10/2003 with that on 4/10/2003. Comparing the yield curve on 3/10/2003 with that for 4/10/2003, it can be seen that the part of the yield curve between year ten and year 20 on 3/10/2003 was 10BP steeper than that on 4/10/2003, while the part of the yield curve between year 20 and year 25 on 3/10/2003 was almost the same as that on 4/10/2003. Due to these differences in the shape of the yield curve, the deviation of the values of (3) and (2) on 3/10/2003 (1 Yen, 40 Sen) was larger than that on 4/10/2003 (1 Yen, 9 Sen). On the other hand, comparing the yield curve on 7/10/2003 with that on 4/10/2003, one may see that the part of the yield curve between year 20 and year 25 on 7/10/2003 was 10BP steeper than that on 4/10/2003, while the part of the yield curve between year ten and year 20 on 7/10/2003 was almost the same as that on 4/10/2003. Due to these differences in the shape of the yield curve, the deviation of the values of (3) and (2) on 7/10/2003（57 Sen) was smaller than that on 4/10/2003 (1 Yen, 9 Sen).

To analyze the effect of the floor level, I compare the value of (3) with the value of (4). The floor level in the value of (3) is 0%, while the floor level in the value of (4) is 0.05%. Thus, if this floor level difference is fully priced into the valuation, the difference in the valuations should reach 50 Sen.

However, on the three issuance dates 3/10/2003, 4/10/2003 and 7/10/2003, the price differences were only 6 Sen, 7 Sen and 3 Sen, respectively. These differences were far less than 50 Sen, and it can be seen that the floor options on the three dates were far out of the money. A closer look at these price differences reveals that that the difference on 4/10/2003, when the yield level was the lowest, was at its highest at 7 Sen. The difference on 3/10/2003, when the yield level was slightly higher than that on 4/10/2003, was 6 Sen, a little less than the difference on 4/10/2004. Furthermore, the difference on 7/10/2003, when the yield level was much higher than that on 4/10/2003, was only 3 Sen, less than half the difference on 4/10/2004. Based on this analysis, it can be seen that the moneyness of the floor option was reasonably reflected in these differences, even though they were far out of the money.

Next, I examine the value of the American option embedded in the retail JGB. Note first that the value of the American option derives from the accumulated reinvestment rate. When the JGB market declines and the yield curve becomes flatter, the accumulated reinvestment rate increases, and the price of the retail JGB is less than the strike price, or, in other words, less than the amount that was received at sale prior to maturity, which was defined in Section 2.2. In this situation, without a put option, in order to enjoy the high reinvestment rate, one must sell the retail JGB at a market price below the strike price. On the other hand, in the case of a put option, one could enjoy a high reinvestment rate without a substantial loss of the capital of the retail JGB, and this is the origin of the value of the American option embedded in the retail JGB.

To analyze the value of the American put option, using the information in Table 1, I compare the value of (4) with the value of (5); I also compare the value of (4) with the value of (6). The former constitutes a comparison of the value of the ten-year floating-rate JGB with a 0.05% floor, without an American option, with the value of the retail JGB under the same αBP. The latter constitutes a comparison of the value of the ten-year floating-rate JGB, with a 0.05% floor, without an American option, and using αBP as in (1), with the value of the retail JGB using 80BP as αBP. On the three issuance dates 3/10/2003, 4/10/2003 and 7/10/2003, the values of (5) were higher than the values of (4) by 2 Yen, 70 Sen; 2 Yen, 42 Sen; and 77 Sen, respectively, and the values of (6) were higher than the values of (4) by 4 Yen, 63 Sen; 7 Yen, 84 Sen; and 83 Sen, respectively. Due to some American option values, the values of (5) and (6) were clearly higher than the value of (4). However, at first glance, the relation between the value of (5) and the value of (6) is a bit strange, because the values of (6) that had small future coupon cash flows, due to the large magnitude of αBP (80BP), were larger than the values of (5) that had large future coupon cash flows, due to the small magnitude of αBP (63BP, 54BP, 66BP). This strange relationship is due to the moneyness of the American put option.

For the three issuance dates 3/10/2003, 4/10/2003 and 7/10/2003, and for αBP between 0BP and 150BP, Figures 11 through 13 illustrate the valuations of: (2) the wholesale JGB, (3) the ten-year floating-rate JGB with a 0% floor, without an American option JGB, and (5) the retail JGB, all as derived using the simulation-based model. The values of (5) at 80BP were simply the values of (6) given in Table 1. Focusing on the values of (3) and (5), as presented in the figures, it may be seen that there are a few differences among the levels of the values, but that the shapes of the graphs are almost the same. In the range of small αBP (less than 30BP), the values of (3) and (5) are very close, and both of them decrease when αBP increases. In the range of intermediate αBP (from 30BP to 70BP), the value of (3) begins to deviate from the value of (5). While the value of (3) still decreases with increased αBP, the value of (5) stays nearly flat as αBP increases. In the range of relatively large αBP (from 70BP to around 100BP), while the value of (3) continues to decrease as αBP increases, the value of (5) rapidly increases with increases in αBP. In the case of extremely large αBP (beyond 100BP), both values decrease when αBP increases.

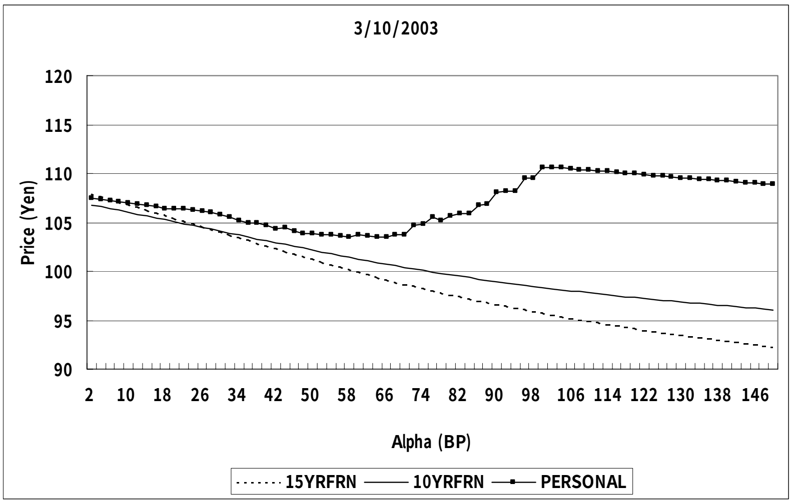

Figure 11：Price sensitivity with respect to αBP at first issuance date 3/10/2003.

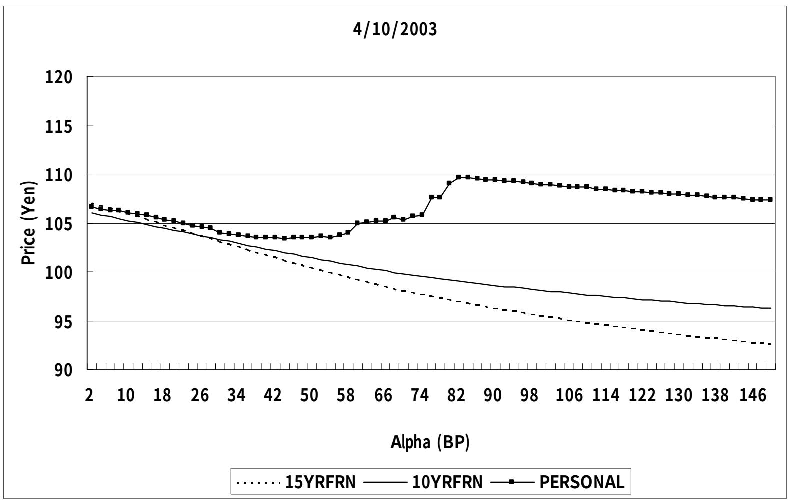

Figure 12：Price sensitivity with respect to αBP at second issuance date 4/10/2003.

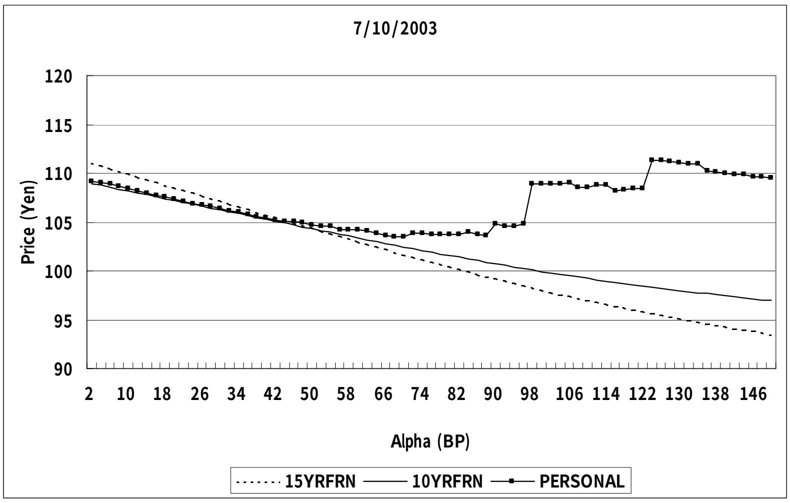

Figure 13：Price sensitivity with respect to αBP at third issuance date 7/10/2003.

These results can be explained as follows. Because the amount that is received from sale prior to maturity is the "face value + accrued interest－two times the coupon amount just prior to the sale," the strike price is less than 100. In the range of small αBP, the expected future coupon cash flow is very large, and the value of the retail JGB without the American option is far above 100 Yen. Therefore, in short, the American option in this αBP range is an out-of-the-money option, and the option retains almost no intrinsic value. The smaller the αBP, the smaller the moneyness of the option. Thus, in the range of small αBP, the value of (3) that does not include the American option is very close to the value of (5), including the American option. The fact that, in the range of intermediate αBP, the value of (5) stays at the same level is due to the balancing effect caused by the increasing value of αBP between the decreasing value of the expected future coupon cash flow and the increasing intrinsic value of the American option. In the range of relatively large αBP, the value of (5) rapidly increases with increases in αBP, because the value of (3) (the total value of the future coupon cash flow) falls below 100 Yen, and the American put option changes from out-of-the money to in-the-money. More precisely, the effect of the increasing intrinsic value of the American option dominates the effect of the decreasing value of the future coupon cash flow. In the range of extremely large αBP, the value of (5) decreases with increases in αBP, because the optimal time of exercise of the American option is already the earliest possible time for this range, and the increasing αBP only causes a decrease in the value of the expected future coupon cash flow.

In order to justify the above factor analysis, I take the results for 3/10/2003 as an example. The five early exercise boundaries in the cases that αBP was equal to 20BP, 40BP, 60BP, 80BP and 100BP are shown in Figure 14, and the five distributions of time hitting the early exercise boundaries are provided in Figures 15 through 19. The early exercise boundaries in Figure 14 indicate that the levels of the boundaries were almost the same when the time of exercise was later than 4.5 years; however, when the time of exercise was earlier than 4.5 years, the levels of the boundaries for large αBP (especially 100BP) were higher than those for small αBP. The higher exercise boundary in this case was caused by the fact that the American option was deep in the money when αBP was 100BP. The moneyness of the American option has been discussed in the analysis above. Therefore, I now examine the moneyness of the American option using the distribution of time hitting the early exercise boundaries. In Figures 15 through 19, a time of exercise of 10 means that the American option was not exercised and that the retail JGB was held to maturity. In accordance with the increase in αBP from 20BP to 100BP (Figures 15-19), the frequencies with which the American option was not exercised decreased from slightly less than 500 to around 200. A closer examination of these figures indicates that, even in the case of the early exercise of the American option, when αBP was as small as 20BP, the time of early exercise was almost surely 4.5 years. Furthermore, one can see that, with the increase of αBP, the time of early exercise became earlier and that, when αBP was as large as 100BP, the time of early exercise fell to the earliest time one year subsequent to the time of issuance.

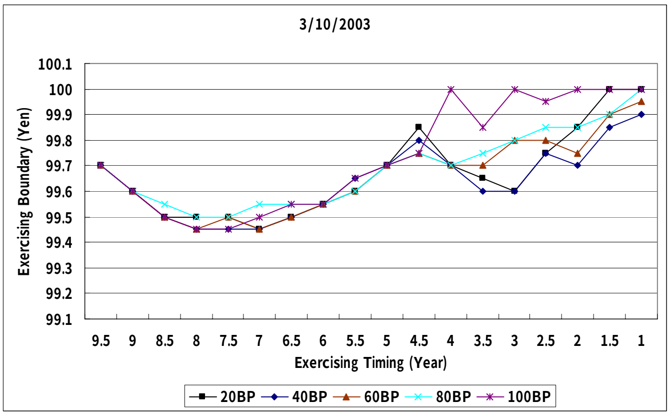

Figure14：On 3/10/2003, the five early exercise boundaries in the cases that αBP was equal to 20BP, 40BP, 60BP, 80BP and 100BP are shown.

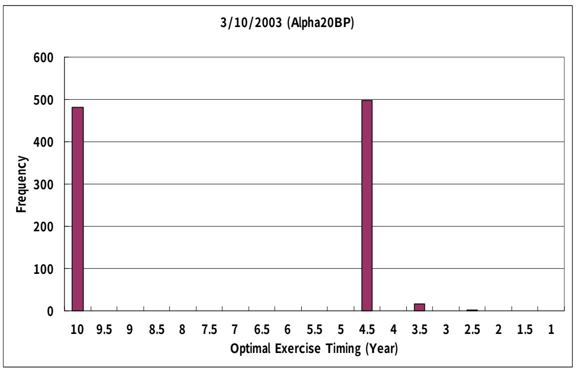

Figure 15：The distribution of time hitting the early exercise boundary (α=20BP)at first issuance date 3/10/2003.

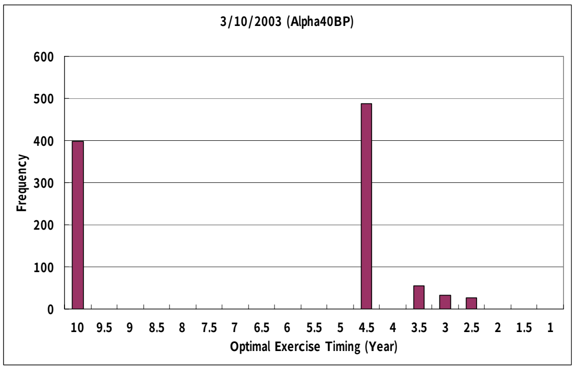

Figure 16：The distribution of time hitting the early exercise boundary (α =40BP) at first issuance date 3/10/2003.

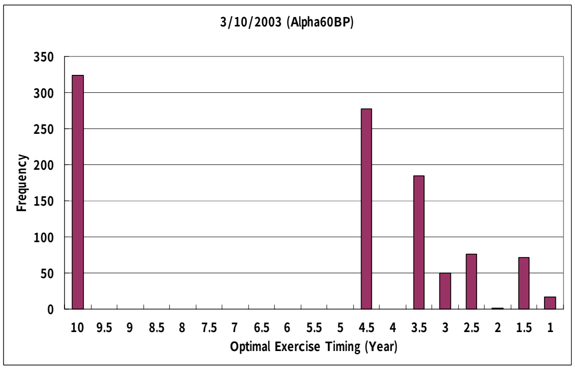

Figure 17：The distribution of time hitting the early exercise boundary (α =60BP) at first issuance date 3/10/2003.

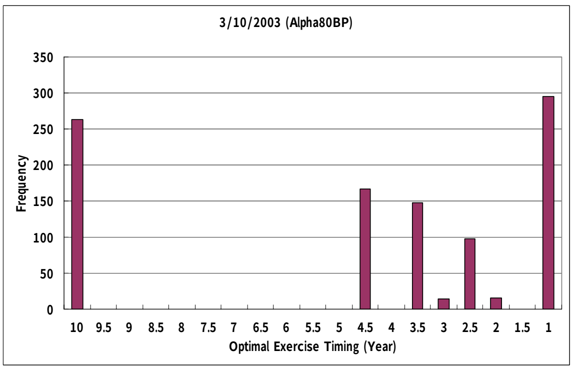

Figure 18：The distribution of time hitting the early exercise boundary (α =80BP) at first issuance date 3/10/2003.

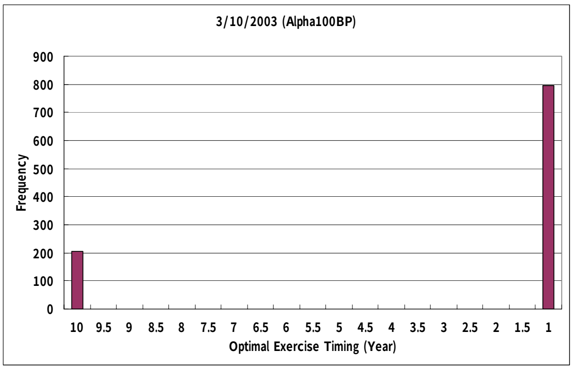

Figure 19：The distribution of time hitting the early exercise boundary (α=100BP) at first issuance date 3/10/2003.

8. Summary and directions for future research

In this study, the primary goal was a clarification of the economic attributes of the ten-year floating-rate retail JGB, which has been issued since 3/10/2003. First, in the course of reaching this goal, the standard industry valuation formula of the wholesale JGB, which is a product similar to the retail JGB, was presented, and the difficulty in applying its valuation formula to the retail JGB was mentioned. Second, some approximation techniques necessary for the use of the forward rate market model were summarized, and the convexity correction method used in the simulation-based model was provided by way of a numeraire change, along with a comparison of it with the standard industry convexity correction method. Third, the valuation model for the retail JGB was presented, and the technique used for parameter estimation was also provided. Finally, in numerical examples, the economic attributes of the retail JGB were compared with those of the wholesale JGB and other similar products. The numerical examples indicated that, in the range of small αBP, the value of the retail JGB was very close to the values of other similar products, and both of them were decreasing in αBP. In the range of intermediate αBP, the value of the retail JGB remained the same and was increasing with αBP, while, in the range of relatively large αBP, the value of the retail JGB was rapidly increasing in αBP. Thus, in these ranges, the graph of the price of the retail JGB with respect to αBP is quite different from those of the other, similar products considered. In the range of extremely large αBP, the prices of both the retail JGB and the other similar products decreased with increases in αBP. It is rather interesting that the above differences in price come from the trade-off, resulting from increases in α BP, between the decreasing value of the expected future coupon cash flow and the increasing magnitude of the intrinsic value of the American option.

Two directions are suggested as avenues for future research. The first involves an increase in the precision of the evaluation of the American option, without a rapid increase in the computational burden of the risk-neutral drift approximations. The second direction involves the incorporation of the smile effect into the evaluation. In the future, I will attempt to make better use of the information available regarding not only the implied in-the-money swaption volatilities but also the implied out-of-the-money swaption volatilities.

In closing, it is my hope that this study may be of some use to practitioners attempting to understand the characteristics of the ten-year floating-rate retail JGB, as well as to those who are interested in applications of the forward rate market model.

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